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NON-PERTURBATIVE QUARKONIUM DISSOCIATION IN HADRONIC MATTER

D. Kharzeev^a, L. McLerran^b and H. Satz^a

^a Theory Division, CERN, CH-1211 Geneva, Switzerland
and

Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany

^b School of Physics and Astronomy, University of Minnesota, MN 55455 Minneapolis, USA

Abstract

We calculate the dissociation rates of quarkonium ground states by tunnelling and direct thermal activation to the continuum. For hadronic matter at temperatures $T \leq 0.2$ GeV, neither of these mechanisms leads to a sufficiently large dissociation to explain the experimentally observed suppression of charmonium. Dissociation by sequential excitation to excited energy levels, although OZI-forbidden, requires further analysis.

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For sufficiently heavy quarks, the dissociation of quarkonium states by interaction with light hadrons can be fully accounted for by short-distance QCD [1–4]. Perturbative calculations become valid when the space and time scales associated with the quarkonium state, r_Q and t_Q , are small in comparison to the nonperturbative scale $\Lambda_{\text{QCD}}^{-1}$

$$r_Q \ll \Lambda_{\text{QCD}}^{-1}, \quad (1a)$$

$$t_Q \ll \Lambda_{\text{QCD}}^{-1}; \quad (1b)$$

$\Lambda_{\text{QCD}}^{-1}$ is also the characteristic size of the light hadrons. In the heavy quark limit, the quarkonium binding becomes Coulombic, and the spatial size $r_Q \sim (\alpha_s m_Q)^{-1}$ thus is small. The time scale is by the uncertainty relation given as the inverse of the binding energy $E_Q \sim m_Q$ and hence also small. For the charmonium ground state J/ψ , we have

$$r_\psi \simeq 0.2 \text{ fm} = (1 \text{ GeV})^{-1} \quad (2)$$

and

$$E_\psi = 2M_D - M_\psi \simeq 0.64 \text{ GeV}. \quad (3)$$

With $\Lambda_{\text{QCD}} \simeq 0.2 \text{ GeV}$, the inequalities (1) seem already reasonably well satisfied, and also the heavy quark relation $E_\psi = (1/m_c r_\psi^2)$ is very well fulfilled. We therefore expect that the dissociation of J/ψ 's in hadronic matter will be governed by the J/ψ -hadron break-up cross section as calculated in short-distance QCD.

Nevertheless, in view of the finite charm quark mass, it makes sense to study possible non-perturbative contributions, in particular for the calculation of the break-up process in matter, where in addition thermal activation may come into play. For an isolated J/ψ -hadron system, non-perturbative interactions can be pictured most simply as a quark rearrangement. Consider putting a J/ψ “into” a stationary light hadron; the quarks could then just rearrange their binding pattern to give rise to transitions such as $J/\psi + N \rightarrow \Lambda_c + \bar{D}$ or $J/\psi + \rho \rightarrow D + \bar{D}$ (Fig. 1). The probability for such a transition can be written as

$$P_{\text{rearr}} \sim \int d^3r R(r) |\phi_\psi(r)|^2, \quad (4)$$

where the spatial distribution of the $c\bar{c}$ bound state is given by the squared wave function $|\phi_\psi(r)|^2$. The function $R(r)$ in eq. (4) describes the resolution capability of the colour field inside the light hadron. Its wave length is of order $\Lambda_{\text{QCD}}^{-1}$, and so it cannot resolve the charge content of very much smaller bound states; in other words, it does not “see” the heavy quarks in a bound state of radius $r_Q \ll \Lambda_{\text{QCD}}^{-1}$ and hence cannot rearrange bonds. The resolution $R(r)$ will approach unity for $r\Lambda_{\text{QCD}} \gg 1$ and drop very rapidly with r for $r\Lambda_{\text{QCD}} < 1$, in the functional form

$$R(r) \simeq (r\Lambda_{\text{QCD}})^n, \quad r\Lambda_{\text{QCD}} < 1, \quad (5)$$

with $n = 2$ [5] or 3 [1]. As a result, the integrand of eq. (4) will peak at some distance r_0 , with $r_Q < r_0 < \Lambda_{\text{QCD}}$. Since the bound state radius of the quarkonium ground state decreases with increasing heavy quark mass, while $R(r)$ is m_Q -independent, $r_0 \rightarrow r_Q \rightarrow$

0 as $m_Q \rightarrow \infty$. Hence P_r vanishes in the limit $m_Q \rightarrow \infty$ because $R(r_0)$ does, indicating that the light quarks can no longer resolve the small heavy quark bound state.

In a potential picture, the situation just described means that the charm quarks inside the J/ψ have to tunnel from $r = r_\psi$ out to a distance at which the light quarks can resolve them, i.e., out to some $r \simeq c\Lambda_{\text{QCD}}^{-1}$, where c is a constant of order unity (Fig. 2). Such tunneling processes are therefore truly non-perturbative: they cover a large space-time region, of linear size $\Lambda_{\text{QCD}}^{-1}$, and do not involve any hard interactions. The first aim of this letter is to estimate the contribution of non-perturbative tunnelling to the dissociation of quarkonium states. Following this, we turn to matter at finite temperature, where the J/ψ can be excited into the continuum by thermal activation. Our second objective is to calculate the rate of dissociation by this mechanism.

In general, the problem of quark tunneling cannot be solved in a rigorous way, since it involves genuine non-perturbative QCD dynamics. However, the large mass of the heavy quark allows a very important simplification, the use of the quasiclassical approximation. In this approximation, the rate of tunneling R_{tun} can be written down in a particularly transparent way: it is simply the product of the frequency ω_ψ of the heavy quark motion in the potential well and the tunnelling probability P_{tun} when the quark hits the wall of the well,

$$R_{\text{tun}} = \omega_\psi P_{\text{tun}} \quad (6)$$

The frequency ω_ψ is determined by the gap to the first radial excitation,

$$\omega_\psi \simeq (M_{\psi'} - M_\psi) \simeq E_\psi. \quad (7)$$

Consider now the potential seen by the $c\bar{c}$ (Fig. 2). For a particle of energy E , the probability of tunneling through the potential barrier $V(r)$ is obtained from the squared wave function in the “forbidden” region. It can be expressed in terms of the action W calculated along the quasiclassical trajectory,

$$P_{\text{tun}} = e^{-2W}, \quad (8)$$

where

$$W = \int_{r_1}^{r_2} |p| dr, \quad (9)$$

Here the momentum $|p|$ is given by

$$|p| = [2M(V(x) - E)]^{1/2}, \quad (10)$$

and r_1, r_2 are the turning points of the classical motion determined from the condition $V(r_i) = E$.

In our case, the width of the barrier is approximately $0.6 \Lambda_{\text{QCD}}^{-1}$, while its height $(V - E)$ is equal to the dissociation threshold E_Q . The mass M in Eq.(10) is the reduced mass, $M = m_Q/2$. We thus have

$$W \simeq 0.6 \sqrt{m_Q E_Q} / \Lambda_{\text{QCD}}. \quad (11)$$

For the J/ψ , we get from (11) the value $W \simeq 3$; this *a posteriori* justifies the use of quasiclassical approximation, which requires $S > 1$.

Using eq. (11), we obtain as final form for the tunneling rate (6)

$$R_{\text{tun}} = E_\psi \exp - (1.2 \sqrt{m_c E_\psi} / \Lambda_{\text{QCD}}). \quad (12)$$

With the above mentioned J/ψ parameters, this leads to the very small dissociation rate

$$R_{\text{tun}} \simeq 9.0 \times 10^{-3} \text{ fm}^{-1}. \quad (13)$$

In terms of R_0 , the J/ψ survival probability is given by

$$S_{\text{tun}} = \exp \left\{ - \int_0^{t_{\text{max}}} dt R_{\text{tun}} \right\}, \quad (14)$$

where t_{max} denotes the maximum time the J/ψ spends adjacent to the light hadron. In the limit $t_{\text{max}} \rightarrow \infty$, S_ψ vanishes. However, the uncertainty relations prevent a localisation of the two systems in the same spatial area for long times. From $\Delta x \leq \Lambda_{\text{QCD}}^{-1}$ we get $\Delta p \geq \Lambda_{\text{QCD}}$, so that the longest time which the J/ψ can spend in the interaction range of the light hadron is

$$t_{\text{max}} = \Lambda_{\text{QCD}}^{-1} \left(1 + \frac{m^2}{\Lambda_{\text{QCD}}^2} \right)^{1/2}, \quad (15)$$

with m for the mass of the light hadron. For nucleons or vector mesons, this time is 4 - 5 fm, and with this, the survival probability is very close to unity, so that tunnelling cannot result in a noticeable non-perturbative J/ψ dissociation.

In a medium at finite temperature, however, the J/ψ can in addition be thermally excited into the continuum and thus become dissociated; this thermal activation could be non-perturbative. Here we shall simply consider a J/ψ in a thermal medium of temperature T and calculate its excitation rate, without asking how the constituents of this medium bring the ground state $c\bar{c}$ into the continuum. Since hadrons in a medium of temperature T may not be able to interact sufficiently hard with the J/ψ to overcome the mass gap to the continuum, we obtain in this way an upper bound to direct thermal dissociation.

The partition function of the system at finite temperature is

$$Z(T) = \sum_n e^{-E_n/T} = Z_\psi(T) + Z_{\text{cont}}(T), \quad (16)$$

where Z_{cont} and Z_ψ are the continuum and the bound-state contributions, respectively. We assume that the $c\bar{c}$ will be distributed among the ground state J/ψ and the continuum above $E = 2M_D$ according to

$$\rho(E, T) = c(E) e^{-E/T} [\delta(1 - E/M_\psi) + \Theta(E - 2M_D)], \quad (17)$$

where $c(E)$ denotes the degeneracy factor, which we take to be constant, $c(E) = c$. The continuum part of the partition function is given by

$$Z_{\text{cont}}(T) = V c \int \frac{d^3 p}{(2\pi)^3} e^{-E/T} \Theta(E - 2M_D), \quad (18)$$

where V is the canonical volume containing the system. Changing variables to

$$p^2 dp = M \sqrt{2ME} dE, \quad (19)$$

we get

$$Z_{\text{cont}}(T) = V \bar{c} \frac{4\pi}{(2\pi)^3} M \sqrt{\frac{\pi M}{2}} T^{3/2} e^{-E_\psi/T}, \quad (20)$$

while the bound-state part is obtained from eq. (17) as

$$Z_\psi(T) = \bar{c}, \quad (21)$$

with $E_\psi = 2M_D - M_\psi$ and $\bar{c} \equiv c e^{-M_\psi/T}$.

The rate of escape into the continuum can be estimated by the average time needed to leave the spatial region of the potential well,

$$R_{\text{act}} = \langle t \rangle^{-1} = \langle v(E) \rangle / L. \quad (22)$$

Here $v(E)$ is the velocity of the (reduced) charm quark in the continuum, and $L \simeq (1 - r_\psi \Lambda_{\text{QCD}}) \Lambda_{\text{QCD}}^{-1}$ is the distance from the average J/ψ radius to the top of the potential well. Changing variables from the overall energy to the energy above M_ψ and using $v(E) = p(E)/M$, we bring eq. (22) into the form

$$R_{\text{act}} = \frac{1}{Z(T)} \frac{V}{ML} \int \frac{d^3 p}{(2\pi)^3} p [\bar{c} \Theta(E - E_\psi) e^{-E/T}]. \quad (23)$$

With the substitution $p(E) = \sqrt{2M(E - E_\psi)}$ we then have after integration the result

$$R_{\text{act}} = \frac{1}{Z(T)} \frac{V}{L} \left(\frac{\bar{c}}{\pi^2} M T^2 \right) e^{-E_\psi/T}, \quad (24)$$

where $V = L^3$ is the canonical volume.

For high temperatures, $T \gg E_\psi$, the continuum gives the dominant contribution to the statistical sum, so that we can replace $Z(T)$ by Z_{cont} in eq. (24) to get

$$R_{\text{act}} = \frac{4}{L} \sqrt{\frac{T}{2\pi M}}. \quad (25)$$

Recalling that the thermal velocity of a free particle in three dimensions is just $v_{\text{th}}(T) = 4\sqrt{T/2\pi M}$, we thus recover the classical high-temperature limit for the thermal activation rate

$$R_{\text{act}} = \frac{v_{\text{th}}(T)}{L}. \quad (26)$$

At low temperatures, for $T \ll E_\psi$, the discrete term in ρ give the main contribution to $Z(T)$. In this temperature range, which is the one of interest here, we thus obtain from eqs. (24) and (21)

$$R_{\text{act}} = \frac{(LT)^2}{3\pi} M e^{-E_\psi/T}, \quad (27)$$

as our final result.

The explicit appearance of the volume factors L in the above results for thermal activation may at first sight seem strange. Its origin is the fact that although the intrinsic Boltzman factor for excitations to the continuum is small, the density of states there is quite large. Nevertheless, the limit of large L is well defined, since for $L \rightarrow \infty$ the continuum part (20) of the partition function dominates, so that the result (27) is replaced by the classical formula (25). The heavy quark limit $M \rightarrow \infty$ is also well defined, since at very large M the size of the system shrinks to $L \sim 1/M$. It is moreover important to remember that for heavy quark-antiquark systems the binding energy (on which the rate (27) depends exponentially) increases with the mass of the quark.

With the same charmonium parameters as above ($E_\psi = 0.64$ GeV, $L = 0.6 \Lambda_{\text{QCD}}^{-1}$, $2M = m_c = 1.5$ GeV) and at the temperature $T = 0.2$ GeV, we thus obtain

$$R_{\text{act}} \simeq 6 \times 10^{-3} \text{ fm}^{-1}, \quad (28)$$

which is of the same size as the tunnelling rate (13). The precise value of L is somewhat uncertain, of course. We feel, however, that $L \simeq 1$ fm is an upper bound, leading to $R_{\text{act}} \leq 2 \times 10^{-2} \text{ fm}^{-1}$ as upper bound for the rate. Thus direct thermal activation can also not provide a significant amount of J/ψ dissociation at temperatures below the binding energy. Already for static media of 4-5 fm life-time the survival probability is very close to unity; any expansion would further reduce activation effects.

We should however be cautious about the application of the thermal activation formula used above. Because of the large density of states in the continuum, the treatment of the factor L was somewhat heuristic. To better understand this dependence, it will probably be necessary to investigate directly the sequential transitions from the ground state to excited states and then to the continuum. Thus the J/ψ could be dissociated by first exciting it to a χ_c and then bringing the χ_c to the continuum. Since the phase space for the continuum is so large, the probability of de-excitation back to a bound state is negligible. Unfortunately, it is difficult to obtain a simple estimate of the various processes which contribute to this sequential excitation. The analysis of this process awaits further work.

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References

- 1) M. E. Peskin, Nucl. Phys. B156 (1979) 365;
G. Bhanot and M. E. Peskin, Nucl. Phys. B156 (1979) 391.
- 2) M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. 65B (1976) 255;
V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B136 (1978) 125.
- 3) A. Kaidalov, in *QCD and High Energy Hadronic Interactions*, J. Trân Thanh Vân (Ed.), Edition Frontieres, Gif-sur-Yvette, 1993.
- 4) D. Kharzeev and H. Satz, Phys. Lett. B 334 (1994) 155.
- 5) F. E. Low, Phys. Rev. D12 (1975) 163;
S. Nussinov, Phys. Rev. Lett. 34 (1975) 1268;
J. F. Gunion and H. Soper, Phys. Rev. D15 (1977) 2617.

Figure Captions

Fig. 1 Rearrangement transition $\rho + J/\Psi \rightarrow D + \bar{D}$.

Fig. 2 J/Ψ dissociation by tunneling.